

Quark matter as dark matter in modeling galactic halo

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Considering the flat rotation curves as input and treating the matter content in the galactic halo region as quark matter, we have found out a background spacetime metric for the region of the galactic halo. We obtain fairly general conditions that ensure that gravity in the halo region is attractive. We also investigate the stability of circular orbits, along with a different role for quark matter. Bag-model quark matter meeting these conditions therefore provides a suitable model for dark matter.

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I. INTRODUCTION

The flatness of the galactic rotation curves of neutral hydrogen clouds in the outer regions of galaxies has led to the hypothesis that galaxies and even clusters of galaxies are pervaded by dark matter [1–3]. To explain the observed constant velocity, it is assumed that the decrease in the energy density is proportional to $1/r^2$, where r is the distance from the center of the galaxy.

In special connection to these galactic halo and dark matter we would like to mention some of the previous works in the sequel as follows. In an investigation under the framework of brane-world models it has been shown by Rahaman et al. [4] that the observed rotation curves result solely from non-local effects of gravitation, such as dark radiation and dark pressure, and does not invoke any exotic matter field. Nandi et al. [5] have considered several aspects of the 4d imprint of the 5d bulk Weyl radiation by combined measurements of rotation curve and lensing effect. In another case the energy density attached to noncommutative geometry has been assumed to diffuse throughout a region which is responsible for producing stable circular orbits and attractive gravity [6]. For some other notable works in this line see Refs.

[7–9].

A number of candidates for this dark matter has been put forward [10–12]. In the present paper we propose that quark matter [13–15] is such a candidate, as previously suggested by several workers [16–18]. The fundamental particles of quark matter do not ordinarily exist as free particles since they are bound together by the strong interaction. However, quark matter is believed to exist at the center of *neutron stars* [16], in *strange stars* [19–22], or even as small pieces of *strange matter* [23, 24].

Regarding the origin and survival of quark matter, it is believed that the Universe underwent a quark gluon phase transition a few microseconds after the big bang [25–27]. In fact, in 1984, Witten [28] proposed that such a transition at a critical temperature $T_C \equiv 100\text{--}200$ MeV could have led to the formation of quark nuggets made of u, d, and s quarks at a density that would be larger than normal nuclear matter density. Laboratory experiments (such as CERN LHC) with relativistic nuclei aim to recreate the conditions similar to those encountered before and in the early hadronisation period [29]. As the expanding Universe cools, the hot quark-gluon-plasma (QGP) freezes slowly into individual hadrons [30].

The survival of such nuggets has been the subject of many investigations [31–34]. According to Bhat-tacharyya et al. [25], not only does a large number of stable quark nuggets exist in the present Universe, but quark matter could be a viable candidate for cosmological dark matter, as already noted. Quantum Chromo Dynamics (QCD), the most accepted theory of strong interaction predicts that under extreme condition a hadronic system can undergo a phase transition from color confined

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hadronic matter to the QGP phase. In Astrophysics, it is speculated that the core of neutron stars may consist of cold QGP [35].

Since we are taking the flat rotation curves as input, the circular orbit is assumed accordingly. The problem is thereby reduced to usages of the MIT bag model [36] to determine conditions ensuring that gravity in the halo region is attractive. Under those conditions, quark matter may be considered as one of the suitable candidates for dark matter. This is the motivation behind the present work.

The investigations are organized as follows: in Sec. 2 we provide basic equations whereas in Sec. III we have considered the galactic rotation curves as an input. The solutions of the equations have been found out for the cases (i) non-interacting two-fluid model, and (ii) interacting two-fluid model in Section 4. In Secs. 5, 6 and 7 we have addressed the question of an attractive gravity for structure formation in the galactic region, stability of circular orbits and dual role of quark matter, respectively. Sec. 8 is devoted for some concluding remarks on the model.

II. BASIC EQUATIONS

In this paper the metric for a spherically symmetric spacetime is taken to be

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 d\Omega^2, \quad (1)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. We are using here geometrized units in which $G = c = 1$.

The energy-momentum tensor of the two-fluid model is given by

$$T_0^0 \equiv \rho_{eff} = \rho + \rho_q, \quad (2)$$

$$T_1^1 = T_2^2 \equiv -p_{eff} = -(p + p_q), \quad (3)$$

In the above two equations (2) and (3) ρ and p correspond to the respective energy density and pressure of the baryonic matter, whereas ρ_q and p_q to the respective dark energy density and pressure due to quark matter. The left-hand sides of equations (2) and (3) are the effective energy density and pressure, respectively, of the composition.

The Einstein field equations are listed next:

$$8\pi(\rho + \rho_q) = e^{-\lambda} \left(\frac{\lambda'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2}, \quad (4)$$

$$8\pi(p + p_q) = e^{-\lambda} \left(\frac{\nu'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2}, \quad (5)$$

$$8\pi(p + p_q) = \frac{e^{-\lambda}}{2} \left[\frac{(\nu')^2 - \lambda'\nu'}{2} + \frac{\nu' - \lambda'}{r} + \nu'' \right], \quad (6)$$

since $T_1^1 = T_2^2$.

In the MIT bag model, the quark matter equation of state (EoS-1) has the simple linear form

$$p_q = \frac{1}{3}(\rho_q - 4B), \quad (7)$$

where B , the bag constant, is in units of $MeV/(fm)^3$ [36, 37].

For normal matter, we use the following equation of state (EoS-2):

$$p = m\rho, \quad (8)$$

where $0 < m < 1$.

Since we are assuming the pressure to be isotropic, therefore, the conservation equation is

$$\frac{d(p_{eff})}{dr} + \frac{1}{2}\nu'(\rho_{eff} + p_{eff}) = 0. \quad (9)$$

III. GALACTIC ROTATION CURVES

The observed flat rotation curves are often considered as evidence for the existence of dark matter. In such galaxies the neutral hydrogen clouds, observed at large distances from the center, are treated as test particles moving in circular orbits due to the gravitational effects of the halo. To derive the tangential velocity of such circular orbits, we start with the line element (1) and then observe that the Lagrangian for a test particle takes the following form:

$$2\mathcal{L} = -e^{\nu(r)}\dot{t}^2 + e^{\lambda(r)}\dot{r}^2 + r^2\dot{\Omega}^2, \quad (10)$$

where as usual $\dot{\Omega}^2 = \dot{\theta}^2 + \sin^2\theta \dot{\phi}^2$. The overdot denotes differentiation with respect to affine parameter s .

Since the metric tensor coefficients do not depend explicitly on t , θ , ϕ , or Ω , the Euler-Lagrange equation yields directly the conserved quantities E and L , the energy and total momentum, respectively: $E = -e^{\nu(r)}\dot{t}$, $L_\theta = r^2\dot{\theta}$, and $L_\phi = r^2\sin^2\theta \dot{\phi}$. So the square of the total angular momentum is $L^2 = L_\theta^2 + (L_\phi/\sin\theta)^2$. It is actually more convenient to use Eq. (10), as considered by Böhmer, Harko and Lobo [38]. Then the total angular momentum becomes $L = r^2\dot{\Omega}$.

With the use of the conserved quantities E and L and the norm of the four-velocity $u^\mu u_\mu = -1$, the geodesic equation becomes

$$-1 = -e^{\nu(r)}\dot{t}^2 + e^{\lambda(r)}\dot{r}^2 + r^2(\dot{\theta}^2 + \sin^2\theta \dot{\phi}^2). \quad (11)$$

Hence we get

$$e^{\nu(r)+\lambda(r)}\dot{r}^2 + e^{\nu(r)} \left(1 + \frac{L^2}{r^2} \right) = E^2. \quad (12)$$

The equation of motion

$$\dot{r}^2 + V(r) = 0, \quad (13)$$

now yields the potential as

$$V(r) = -e^{-\lambda(r)} \left(e^{-\nu(r)} E^2 - \frac{L^2}{r^2} - 1 \right). \quad (14)$$

Following the work of Nucamendi, Salgado and Sudarsky [39], as it is more convenient to use the effective potential $V_{eff}(r)$, we write Eq. (12) in the form

$$e^{\lambda(r)}\dot{r}^2 + 1 + \frac{L^2}{r^2} - e^{-\nu(r)}E^2 = 0. \quad (15)$$

Recall that for the case of circular stable orbits, the effective potential must satisfy the following conditions:

(1) $\dot{r}^2 = 0$, (2) $\frac{\partial V_{eff}}{\partial r} = 0$, and (3) $\frac{\partial^2 V_{eff}}{\partial r^2} > 0$. By using

$$V_{eff}(r) = 1 + \frac{L^2}{r^2} - e^{-\nu}E^2, \quad (16)$$

the first condition gives directly

$$E^2 = e^{\nu} \left(1 + \frac{L^2}{r^2} \right) \quad (17)$$

from Eq. (15), while the second condition yields

$$\frac{L^2}{r^2} = \frac{1}{2} r \nu' e^{-\nu} E^2. \quad (18)$$

These equations can also be rewritten as

$$E^2 = \frac{e^{\nu}}{1 - \frac{1}{2} r \nu'} \quad (19)$$

and

$$L^2 = \frac{\frac{1}{2} r^3 \nu'}{1 - \frac{1}{2} r \nu'}. \quad (20)$$

Before considering the third condition, $V_{eff}(r)_{rr} > 0$, we need to obtain an expression for the tangential velocity v^ϕ . In terms of proper time, this is given by [40]

$$\begin{aligned} (v^\phi)^2 &= e^{-\nu} r^2 \left(\frac{d\Omega}{dt} \right)^2 = e^{-\nu} r^2 \left(\frac{d\Omega}{ds} \frac{ds}{dt} \right)^2 \\ &= e^{-\nu} r^2 \dot{\Omega}^2 \frac{1}{\dot{t}}. \end{aligned}$$

Substituting for $\dot{\Omega}^2$, we get from Eq. (18)

$$(v^\phi)^2 = \frac{e^{\nu} L^2}{r^2 E^2} = \frac{1}{2} r \nu'. \quad (21)$$

This expression can be integrated to yield

$$e^{\nu} = B_0 r^l, \quad (22)$$

where B_0 is an integration constant and $l = 2(v^\phi)^2$. Equivalently, the line element (1) can be written

$$ds^2 = - \left(\frac{r}{r_0} \right)^{2(v^\phi)^2} dt^2 + e^{\lambda(r)} dr^2 + r^2 d\Omega^2. \quad (23)$$

As a result, the model has a well-defined Newtonian limit [38]. (Observe also that $V_{eff}(r) \sim 1/r^2$, characteristic of dark matter, as a consequence of the flat rotation curves.)

Now from Eq. (16)

$$V_{eff}(r)_{rr} = \frac{6L^2}{r^4} - E^2 e^{-\nu} (\nu')^2 + E^2 e^{-\nu} \nu''. \quad (24)$$

By making the substitutions for L^2 , E^2 , and ν , the second derivative reduces to

$$V_{eff}(r)_{rr} = \frac{2l}{r^2} > 0, \quad (25)$$

confirming the existence of stable orbits (a detailed analysis of which will be provided in the Sec. VI).

IV. SOLUTIONS

In this section we determine $e^{\lambda(r)}$ in the line element (1) by considering two cases, a non-interacting two-fluid model and an interacting two-fluid model.

A. Non-interacting two-fluid model

In our first case, the two fluids, normal matter and quark matter, do not interact. The resulting conservation equations are therefore independent of each other. Using Eqs. (7) and (8), we have

$$\frac{d\rho}{dr} + \nu' \left(\frac{1+m}{2m} \right) \rho = 0 \quad (26)$$

and

$$\frac{d\rho_q}{dr} + 2\nu'(\rho_q - B) = 0. \quad (27)$$

The solutions are

$$\rho = \rho_0 r^{-l(1+m)/2m} \quad (28)$$

and

$$\rho_q = B + \rho_{0q} r^{-2l}, \quad (29)$$

where ρ_0 and ρ_{0q} are integration constants. Equations (4), (5), and (6) now yield

$$\begin{aligned} 8\pi [\rho(1+3m) + (2\rho_q - 4B)] &= \\ e^{-\lambda} \left[-\frac{\lambda' \nu'}{2} + \frac{(\nu')^2}{2} + \nu'' + \frac{2\nu'}{r} \right]. \end{aligned} \quad (30)$$

This equation is linear in $e^{-\lambda}$ and leads to

$$\frac{d}{dr} (e^{-\lambda} r^{2+l}) = 8\pi \left(\frac{2}{l} \right) r^{3+l} [\rho(1+3m) + 2\rho_q - 4B] \quad (31)$$

by making use of Eq. (22). Incorporating Eqs. (28) and (29), we now obtain

$$\begin{aligned} e^{-\lambda} &= D r^{-2-l} + \frac{2}{l} 8\pi \left[\frac{\rho_0(1+3m)r^{2-l(1+m)/2m}}{4+l-l(1+m)/2m} \right. \\ &\quad \left. + \frac{2\rho_{0q}r^{2-2l}}{4-l} - \frac{2Br^2}{4+l} \right], \end{aligned} \quad (32)$$

where D is an integration constant.

B. Interacting two-fluid model

In our second case, the two fluids are assumed to interact. The resulting conservation equations will then take the following form:

$$\frac{d\rho}{dr} + \nu' \left(\frac{1+m}{2m} \right) \rho = Q \quad (33)$$

and

$$\frac{d\rho_q}{dr} + 2\nu'(\rho_q - B) = -3Q. \quad (34)$$

The quantity Q expresses the interaction between the dark-energy components. We assume that there is an energy transfer from quark matter to normal matter. So a positive Q is a natural choice that ensures that the second law of thermodynamics is fulfilled [41]. Moreover, Eqs. (33) and (34) imply that the interaction term is proportional to r and vanishes as $r \rightarrow \infty$. To meet these criteria, we assume that

$$Q \propto \frac{1}{r^n} \Rightarrow Q = \frac{Q_0}{r^n}, \quad (35)$$

where $0 < n \leq a$ for some $a < 1$ and Q_0 is a constant of proportionality. (This assumption is made strictly for computational convenience, allowing us to write explicit solutions; so no particular physical significance should be attached to the range on n). The respective solutions of Eqs. (33) and (34) are now given by

$$\rho = \rho_{0(in)} r^{-l(1+m)/2m} + \frac{Q_0 r^{1-n}}{1-n+l(1+m)/2m} \quad (36)$$

and

$$\rho_q = B + \rho_{0q(in)} r^{-2l} - \frac{3Q_0 r^{1-n}}{1-n+2l}, \quad (37)$$

where $\rho_{0(in)}$ and $\rho_{0q(in)}$ are integration constants.

The solution of Eq. (30) now becomes

$$\begin{aligned} e^{-\lambda} = & F r^{-2-l} + \frac{2}{l} 8\pi \left[\frac{\rho_{0(in)}(1+3m)r^{2-l(1+m)/2m}}{4+l-l(1+m)/2m} \right. \\ & \left. + \frac{2\rho_{0q(in)}r^{2-2l}}{4-l} - \frac{2Br^2}{4+l} \right] \\ & + \frac{2}{l} 8\pi \left[\frac{Q_0(1+3m)r^{3-n}}{[1-n+l(1+m)/2m](5-n+l)} \right. \\ & \left. - \frac{6Q_0r^{3-n}}{(1-n+2l)(5-n+l)} \right], \quad (38) \end{aligned}$$

where F is an integration constant. Observe that apart from the integration constants, the first part of solution (38) is the same as solution (32).

V. ON THE QUESTION OF AN ATTRACTIVE GRAVITY

We may consider the question of an attractive gravity in the halo region by studying the geodesic equation

$$\frac{d^2 x^\alpha}{d\tau^2} + \Gamma_{\mu\gamma}^\alpha \frac{dx^\mu}{d\tau} \frac{dx^\gamma}{d\tau} = 0. \quad (39)$$

This equation implies that [6]

$$\frac{d^2 r}{d\tau^2} = -\frac{1}{2} e^{-\lambda} \left[\frac{d}{dr} e^\lambda \left(\frac{dr}{d\tau} \right)^2 + \frac{d}{dr} e^\nu \left(\frac{dt}{d\tau} \right)^2 \right]. \quad (40)$$

Since $\dot{r} = 0$, as before, we get from Eq. (22),

$$\frac{d^2 r}{d\tau^2} = -\frac{1}{2} e^{-\lambda} B_0 l r^{l-1} \left(\frac{dt}{d\tau} \right)^2 < 0 \quad (41)$$

as long as $e^{-\lambda} > 0$, thereby yielding an attractive gravity in the halo region.

In the non-interacting case related to Eq. (32), the dominant terms are the last two because of the size of the bag constant B . We can see from the EoS-1 in Eq. (7) that

$$p_q = \frac{1}{3}(\rho_{0q} r^{-2l} - 3B). \quad (42)$$

So whenever the pressure p_q is positive, $\rho_{0q} > 3Br^{2l} > 3B$. It follows that

$$e_{eff}^{-\lambda} = \frac{2\rho_{0q} r^{2-2l}}{4-l} - \frac{2Br^2}{4+l} \quad (43)$$

is positive and since $\rho_0 \ll \rho_q$ and $l \approx 0.000001$ [8], $e^{-\lambda(r)}$ is also positive.

The same argument can be applied to the interacting case because of the similarity between Eqs. (32) and (38). So we obtain an attractive gravity in both cases.

The conclusion is somewhat different if p_q is negative. Based on Eq. (42), ρ_{0q} can still be large enough to yield $e_{eff}^{-\lambda} > 0$. Now suppose that the corresponding expression in Eq. (38) is also positive. At a first glance it seems difficult to quantify precisely the structure of quark matter as much of it even now remains conjectural. However, for certain combinations of values of the various parameters, the extra terms involving Q_0 will produce a negative value for the total. So we no longer have an attractive gravity for the interacting case. Of course, if the pressure p_q is negative and sufficiently large in absolute value, then $e^{-\lambda(r)} < 0$ in both cases, so that quark matter would not be a suitable model.

VI. ANALYSIS OF THE STABILITY OF CIRCULAR ORBITS

In the Introduction we have assumed the circular orbits as an input of flat rotation curve and have shown stable

via the Eq. (25). Here we analyze the stability of the orbits in a more detailed form to distinguish between the interacting and non-interacting cases. Let a test particle with four velocity $U^\alpha = \frac{dx^\alpha}{d\tau}$ moving in the region of spacetime given in (23). Assuming $\theta = \pi/2$, the equation $g_{\nu\sigma}U^\nu U^\sigma = -m_0^2$ yields

$$\left(\frac{dr}{d\tau}\right)^2 = E^2 + V(r), \quad (44)$$

with

$$V(r) = - \left[E^2 \left(1 - \frac{r^{-l} e^{-\lambda}}{B_0} \right) + e^{-\lambda} \left(1 + \frac{L^2}{r^2} \right) \right]. \quad (45)$$

Here the two conserved quantities, namely relativistic energy (E) and angular momentum (L) per unit rest mass of the test particle respectively are

$$E = \frac{U_0}{m_0} \quad \text{and} \quad L = \frac{U_3}{m_0}. \quad (46)$$

If the circular orbits are defined by $r = R$, then $\frac{dR}{d\tau} = 0$ and, additionally, $\frac{dV}{dr} |_{r=R} = 0$. Above two conditions result

$$L = \pm \sqrt{\frac{l}{2-l}} R \quad (47)$$

and, using L in $V(R) = -E^2$, we get

$$E = \pm \sqrt{\frac{2B_0}{2-l}} R^{l/2}. \quad (48)$$

The orbits will be stable if $\frac{d^2V}{dr^2} |_{r=R} < 0$ and unstable if $\frac{d^2V}{dr^2} |_{r=R} > 0$.

By putting the expressions for L and E in $\frac{d^2V}{dr^2} |_{r=R}$ and then by using the Eq. (38) of the interacting case, finally we get

$$\begin{aligned} \frac{d^2V}{dr^2} |_{r=R} = & - \left[\frac{lR^2}{2-l} \{ F(4+l)(5+l)R^{-6-l} \right. \\ & + F_1 \frac{l(1+m)}{2m} (1 + \frac{l(1+m)}{2m}) R^{-2-\frac{l(1+m)}{2m}} \\ & + F_2 2l(1+2l)R^{-2-2l} + F_4 n(1-n)R^{-1-n} \} \\ & + F(2+l)(3+l)R^{-4-l} \\ & + F_1 (2 - \frac{l(1+m)}{2m}) (1 - \frac{l(1+m)}{2m}) R^{-\frac{l(1+m)}{2m}} \\ & + F_2 (2-2l)(1-2l)R^{-2l} + 2F_3 + F_4 (3-n)(2-n)R^{1-n} \\ & + \frac{2R^l}{2-l} \{ F(2+2l)(3+2l)R^{-4-2l} + \\ & F_1 (2 - \frac{l(1+m)}{2m}) (1 - \frac{l(1+m)}{2m}) R^{-l-\frac{l(1+m)}{2m}} \\ & + F_2 (2-2l)(1-2l)R^{-3l} + \\ & \left. F_3 (2-l)(1-l)R^{-l} + F_4 (3-n-l)(2-l-n)R^{1-n-l} \} \right], \end{aligned} \quad (49)$$

where

$$\begin{aligned} F_1 &= 8\pi \left(\frac{2}{l} \right) \left[\frac{\rho_{0(in)}(1+3m)}{4+l-\frac{l(1+m)}{2m}} \right], \\ F_2 &= 8\pi \left(\frac{2}{l} \right) \left[\frac{2\rho_{0q(in)}}{4-l} \right], \\ F_3 &= 8\pi \left(\frac{2}{l} \right) \left[\frac{2B}{4+l} \right], \\ F_4 &= 8\pi \left(\frac{2}{l} \right) \left[\frac{Q_0(1+3m)}{[1-n+l(1+m)/2m](5-n+l)} - \frac{6Q_0}{(1-n+2l)(5-n+l)} \right]. \end{aligned}$$

We note that for non-interacting case when $Q_0 = 0$, $\frac{d^2V}{dr^2} |_{r=R} < 0$ since, $l \approx 10^{-6}$ and $m, n < 1$. Thus the circular orbits are always stable for non-interacting situation. When, $Q_0 \neq 0$ i.e. for interacting two fluids, $F_4 < 0$. However, if Q_0 is very small then, $\frac{d^2V}{dr^2} |_{r=R}$ is also negative and circular orbits are stable. The last condition implies that the interaction is very weak. This weak interaction is a characteristics of quark matter at higher temperatures. Thus, in the Weakly Interacting Massive Particle (WIMP) category of the dark-matter halo, besides neutrino-like light particles, quark matter may be one of the possible candidates, which, in turn, has important implications for the galactic structure formation and evolution.

VII. BEHAVIOR OF QUARK MATTER: A DUAL ROLE

The above results may have an interesting interpretation on a cosmological scale. If ρ_q and p_q refer to the matter content in the galactic halo region, then, as we have seen, if $p_q > 0$, then bag-model quark matter behaves like dark matter. Now suppose that ρ_q refers to the energy density obtained by including a region beyond the halo large enough so that the resulting reduced value leads to $p_q < 0$ in Eq. (42). Assume also that $l = 2(v^\phi)^2$ decreases so gradually in the outward radial direction that we may treat it as a constant. Substituting the expression for p_q from Eq. (42) in the Friedmann equation $(d^2a(t)/dt^2)/a(t) = -\frac{4\pi}{3}(\rho + 3p)$, we obtain

$$\frac{1}{a(t)} \frac{d^2a(t)}{dt^2} = -\frac{4\pi}{3}(\rho_q + \rho_{0q}r^{-2l} - 3B). \quad (50)$$

Sufficiently small ρ_q and ρ_{0q} now result in a positive acceleration, in the manner of dark energy. It seems therefore on the galactic level, quark matter behaves like dark matter and on the global level like dark energy.

VIII. CONCLUSION

By taking the flat rotation curves as input and treating the matter content in the galactic halo region as quark matter, we obtained a spacetime metric for the galactic halo.

One can see that the solutions given in the Eq. (32) and Eq. (38) are the interior solution of the spacetime metric given by the Eq. (23) which is neither asymptotically flat nor a spacetime due to a centrally symmetric

black hole. The metric describes that region for which the tangential velocity of the test particle is constant.

Since flat rotation curves were assumed, the problem of determining the suitability of quark matter as a model for dark matter was reduced to finding conditions under which gravity in the halo region is attractive.

We used the MIT bag model whose equation of state is $p_q = \frac{1}{3}(\rho_q - 4B)$ and considered two cases, non-interacting and interacting two-fluid models (referring to quark matter and baryonic matter). In both cases, if the pressure $p_q > 0$, gravity in the halo is attractive. If $p_q < 0$, with a sufficiently large absolute value, then gravity in the halo is repulsive. There exist values of p_q between these extremes in which gravity is attractive in the non-interacting case but not in the interacting case.

We have also investigated the stability of circular orbits and shown that the circular orbits are always stable for non-interacting situation. However, interacting two fluids circular orbits are found to be stable under certain conditions which imply that the interaction is very weak.

In our investigation it is also revealed that on the galactic level, quark matter behaves like dark matter whereas on the global level like dark energy.

In summary, however, quark matter fitting the MIT bag model is indeed a suitable model for dark matter

under fairly general conditions.

As an additional remark, in the interacting case between quark and neutral hydrogen a rise of temperature is expected to happen. If this environment of increased temperature in the interstellar medium is detectable observationally then the present model is physically viable one. The aspect of thermodynamical processes involved in this phenomenon can be undertaken in a future project.

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- [1] J. Oort, Bull. Astron. Inst. Neth. VI **249** (1932) 249.
 - [2] F. Zwicky, Helvet. Phys. Acta **6** (1933) 110.
 - [3] F. Zwicky, Astrophys. J. **86** (1937) 217.
 - [4] F. Rahaman, et al., Mon. Not. R. Astron. Soc. **389** (2008) 27.
 - [5] K.K. Nandi, et al., Mon. Not. R. Astron. Soc. **399** (2009) 2079.
 - [6] F. Rahaman et al., Gen. Relativ. Gravit. **44** (2012) 905.
 - [7] T. Matos, F.S. Guzmán and D. Nuñez, Phys. Rev. D **62** (2000) 061301.
 - [8] K.K. Nandi, I. Valitov, and N.G. Migranov, Phys. Rev. D **80** (2009) 047301.
 - [9] F. Rahaman, et al., Phys. Lett. B **694** (2010) 101512.
 - [10] G. Efstathiou, W. Sutherland, and S.J. Madox, Nat. **348** (1990) 705.
 - [11] A.C. Pope, et al., Astrophys. J. **607** (2004) 655.
 - [12] M. Yu. Khlopov, Mod. Phys. Lett. A, **26** (2011) 2823.
 - [13] U. Heinz and M. Jacob, arXiv:nucl-th/0002042.
 - [14] T. Schaefer, in The Proceedings of the BARC Workshop on Quarks and Mesons, arXiv:hep-ph/0304281 (2003).
 - [15] B. Müller, arXiv:nucl-th/0508062.
 - [16] M.A. Perez-Garcia, J. Silk and J.R. Stone, Phys. Rev. Lett. **105** (2010) 141101.
 - [17] M.M. Forbes, K. Lawson and A.R. Zhitnitsky, Phys. Rev. D **82** (2010) 083510.
 - [18] S. Weissenborn et al., Astrophys. J. **740** (2011) L14.
 - [19] J.J. Drake, Astrophys. J. **572** (2002) 996.
 - [20] P. Jaikumar, S. Reddy and A.W. Steiner, Phys. Rev. Lett. **96** (2006) 041101.
 - [21] H. Rodrigues, S. Barbosa Duarte and J.C.T. de Oliveira, Astrophys. J. **730** (2011) 31.
 - [22] G.H. Bordbar and A.R. Peivand, arXiv:1103.1250.
 - [23] J. Madsen, Lect. Notes Phys. **516** (1999) 162.
 - [24] F. Weber, Prog. Part. Nucl. Phys. **54** (2005) 193.
 - [25] A. Bhattacharyya, et al., Nucl. Phys. A **661** (1999) 629.
 - [26] A. Bhattacharyya, et al., Phys. Rev D **61** (2000) 083509.
 - [27] A. Bhattacharyya, et al., Pramana **60** (2003) 909.
 - [28] E. Witten, Phys. Rev D **30** (1984) 272.
 - [29] EMU01 Collab., M.I. Adamovich, et al., Phys. Lett. B **263** (1991) 539.
 - [30] M. Fromerth and J. Rafelski, *CINPP Conference*, Kolkata, February 2005, page 22.
 - [31] J. Madsen, H. Heiselberg and K. Riisager, Phys. Rev. D **34** (1986) 2974.
 - [32] J. Madsen and M. L. Olesen, Phys. Rev. D **43** (1991) 1069.
 - [33] P. Bhattacharjee, et al., Phys. Rev. D **48** (1993) 4630.
 - [34] K. Sumiyoshi and T. Kajino, Nucl. Phys. **24** (1991) 80.
 - [35] T. Walker, et al., Astrophys. J. **376** (1991) 51.
 - [36] A. Chodos, et al., Phys. Rev. D **9** (1974) 3471.
 - [37] F. Rahaman, et al., Eur. Phys. J. C, DOI: 10.1140/epjc/s10052-012-2071-5, arXiv:1108.6125.
 - [38] C.G. Böhm, T. Harko and F. S. N. Lobo, Astropart. Phys. **29** (2008) 386.
 - [39] U. Nucamendi, M. Salgado and D. Sudarsky, Phys. Rev. D **63** (2001) 125016.
 - [40] L.D. Landau and E.M. Lifshitz, *The Classical Theory of Fields*, Oxford, Pergamon Press (1975).
 - [41] D. Pavon and B. Wang, Gen. Relativ. Gravit. **41** (2009) 1.